

# AP Rev. #10 Key

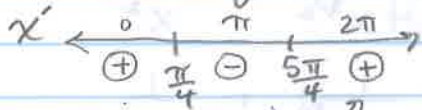
1a)  $x'(t) = e^{-t} \cos t - e^{-t} \sin t$

$$x'(t) = e^{-t}(\cos t - \sin t)$$

$$e^{-t} \neq 0 \quad \cos t = \sin t$$

$$t = \frac{\pi}{4}, \frac{5\pi}{4}$$

Farthest left @ minimum



$$t = \frac{5\pi}{4}$$

b)  $x''(t) = \cancel{e^{-t}(-\sin t)} - e^{-t} \cos t - e^{-t} \cos t + \cancel{e^{-t} \sin t}$

$$x''(t) = -2e^{-t} \cos t$$

$$A(x''(t)) + x'(t) + x(t) = 0$$

$$-2Ae^{-t} \cos t + e^{-t} \cos t - \cancel{e^{-t} \sin t} + \cancel{e^{-t} \sin t} = 0$$

$$e^{-t}(-2A + 1) = 0$$

$$e^{-t} \neq 0 \quad -2A + 1 = 0$$

$$A = \frac{1}{2}$$

2a) using tangent line approx @  $t=5$   
 $(5, 30)$  slope:  $r'(5) = 2 \text{ ft/min}$

$$r - 30 = 2(t - 5)$$

$$r = 2t + 20$$

$$r = 2(5.4) + 20$$

$$r = 30.8 \text{ ft @ } t = 5.4$$

Estimate is greater than actual b/c  $r'(t)$  is decreasing on  $5 < t < 7$  (from table)

b)  $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \frac{dr}{dt} = 2 \text{ (from table)} \quad t=5, r=30$

$$\frac{dV}{dt} = 4\pi(30)^2(2)$$

$$\frac{dV}{dt} = 7200\pi \text{ ft}^3/\text{min}$$

$$3a) f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$$

$$f''(x) = -\frac{k}{4x^{3/2}} + \frac{1}{x^2}$$

$$b) 0 = \frac{k}{2\sqrt{x}} - \frac{1}{x}$$

$$f''(x) = -\frac{2}{4x^{3/2}} + \frac{1}{x^2}$$

$$\frac{1}{x} = \frac{k}{2\sqrt{x}}$$

$$k = \frac{2\sqrt{x}}{x}$$

$$f''(x) = -\frac{1}{2x^{3/2}} + \frac{1}{x^2}$$

$$f''(1) = -\frac{1}{2} + 1$$

$$\text{when } x=1, \boxed{k=2}$$

= positive confirms concave up  
 $\therefore$  relative min @  $x=1$

c) PI on x-axis:

$$\text{PI} \rightarrow 0 = -\frac{k}{4x^{3/2}} + \frac{1}{x^2}$$

x-int from original eq:

$$0 = k\sqrt{x} - \ln x$$

$$\frac{k}{4x^{3/2}} = \frac{1}{x^2}$$

$$\ln x = k\sqrt{x}$$

$$k = \frac{1}{x^2} \cdot 4x^{3/2}$$

$$k = \frac{\ln x}{\sqrt{x}}$$

$$k = \frac{4}{\sqrt{x}}$$

set =

$$\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$$

$$k = \frac{\ln e^4}{\sqrt{e^4}}$$

$$k = \frac{4}{\sqrt{e^4}}$$

$$4 = \ln x$$

$$= \frac{4}{(e^4)^{1/2}}$$

$$x = e^4$$

$$k = \boxed{\frac{4}{e^2}}$$

$$k = \boxed{\frac{4}{e^2}}$$

4) a) PI where  $g'$  goes from incr to decr. or decr to incr.  
 @  $x=1, 4$  (from looking at the graph)

b) abs. max @ endpts or where  $g'$  goes from + to -  
 check:  $x = -3, 2, 7$

from  $x = -3$  to  $x = 2$ ,  $g'$  is more negative than positive which means  $g$  will decrease more than increase so  $g$  will be higher at  $x = -3$  than at  $x = 2$ .  
 By same reasoning,  $g$  is higher @  $x = 2$  than @  $x = 7$   
 So absolute max @  $x = -3$

$$5) a) 2x + 2 + 4y^3 \frac{dy}{dx} + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} \frac{(4y^3 + 4)}{4(y^3 + 1)} = - \frac{2(x+1)}{2 \cdot 4(y^3 + 1)}$$

$$\frac{dy}{dx} = - \frac{(x+1)}{2(y^3 + 1)}$$

$$b) m = \frac{-(-2+1)}{2(1+1)} = \frac{1}{4} \longrightarrow y - 1 = \frac{1}{4}(x + 2)$$

c) vertical tangent where  $\frac{dy}{dx}$  is undef:

$$y^3 + 1 = 0$$

$$y = -1 \rightarrow \text{into original: } x^2 + 2x + (-1)^4 + 4(-1) = 5$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4, 2$$

d) horiz. tangent where

$$\frac{dy}{dx} = 0 \rightarrow x + 1 = 0$$

$$x = -1$$

is there an x-int @ (-1, 0)?

$$(-1)^2 + 2(-1) + 0^4 + 0 \stackrel{?}{=} 5$$

$$1 - 2 \neq 5 \quad \therefore \text{No horiz. tangent @ an x-int.}$$

$$6) L'(5.5) \approx \frac{150 - 126}{7 - 4} = \frac{24}{3} = 8 \text{ people/hr.}$$